

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -2 EXAMINATION- 2023

B. Tech-III Semester (CSE/IT)

COURSE CODE (CREDITS): 18B11MA313/3

MAX. MARKS: 25

COURSE NAME: Probability and Statistics

COURSE INSTRUCTORS: RAD, BKP*, SST

MAX. TIME: 1 Hour 30 Minutes

Note: (a) All questions are compulsory.

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required for solving problems.

(d) Calculator is allowed.

1. A new computer program consists of two modules. The first module contains an error with probability 0.2. The second module is more complex; it has a probability of 0.4 to contain an error, independently of the first module. An error in the first module alone causes the program to crash with probability 0.5. For the second module, this probability is 0.8. If there are errors in both modules, the program crashes with probability 0.9. Suppose the program crashed. What is the probability of errors in both modules?

[5M](CO-1)

2. If the random variables X , Y and Z have the means $\mu_x = 2$, $\mu_y = 4$, $\mu_z = 3$, the variances $\sigma_x^2 = 6$, $\sigma_y^2 = 10$ and $\sigma_z^2 = 14$, and co-variances $\sigma_{xy} = 1$, $\sigma_{xz} = -2$, and $\sigma_{yz} = 3$. Find the mean and covariance of $U = X + 2Y + Z$ and $V = 4X - Y - 2Z$.

[4M](CO-3)

3. Let X and Y be two continuous random variables with joint probability density function

$$f_{XY}(x, y) = \begin{cases} \frac{12}{5}xy(1+y), & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{else} \end{cases}$$

- a) Find the probability $P\left(\frac{1}{4} \leq X \leq \frac{1}{2}, \frac{1}{3} \leq Y \leq \frac{2}{3}\right)$.

- b) Find out whether X and Y are independent.

[2M+2M](CO-3)

4. A program consists of two modules. The number of errors, X , in the first module and the number of errors, Y , in the second module have the joint distribution, $P(0,0) = P(0,1) = P(1,0) = 0.2$, $P(1,1) = P(1,2) = P(1,3) = 0.1$, $P(0,2) = P(0,3) = 0.05$.

Find:

- The marginal distributions of X and Y .
- The probability of no errors in the first module.
- The mean number of errors in the second module. **[2M+1M+1M](CO-3)**

5. Consider a random variable X with mgf $M_X(t) = \left(\frac{2+e^t}{3}\right)^9$. **[4M] (CO-3)**

- What is the probability that X is less than 2?
- Calculate the mean and variance of X .

6. A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours. **[4M] (CO-3)**

- Define the random variable of interest with range of values.
- Find the probability that it will work continuously for 5 hours without a breakdown.
- What is the probability that, during 8 hours, the robot will break down at least once?
