JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATION- 2023

B. Tech.-I Semester (ECE/ECM)

COURSE CODE (CREDITS): 18B11MA314 (4)   MAX. MARKS: 15

COURSE NAME: Probability Theory and Random Processes

COURSE INSTRUCTORS: Saurabh Srivastava   MAX. TIME: 1 Hour

Note: *(a) All questions are compulsory.*

(b) Marks are indicated against each question in square brackets.

(c) The candidate is allowed to make Suitable numeric assumptions wherever required

for solving problems.

(d) Use of scientific calculator is allowed.

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1. Spaceman Spiff's spacecraft has a warning light that is supposed to switch on when the freem blasters are overheated. Let W be the event "the warning light is switched on" and F "the freem blasters are overheated." Suppose the probability of freem blaster overheating $P(F)$ is 0.01, that the light is switched on when they actually *are* overheated is 0.99, and that there is a 2% chance that it comes on when nothing is wrong: $P(W|F^c) = 0.02$.

   a) Determine the probability that the warning light is switched on.

   b) Determine the conditional probability that the freem blasters are overheated, given that the warning light is on. **[2M+2M](CO-1)**

2. If a person visits his dentist, suppose that the probability that he will have his teeth cleaned is 0.44, the probability that he will have a cavity filled is 0.24, the probability that he will have a tooth extracted is 0.21, what is the probability that a person visiting his dentist will have at least one of these treatments? **[2M](CO-1)**

3. If a discrete random variable X has the distribution function

$$F_X(x) = \begin{cases} 0, & x < -1 \\ \frac{1}{4}, & -1 \le x < 1 \\ \frac{1}{2}, & 1 \le x < 3 \\ \frac{3}{4}, & 3 \le x < 5 \\ 1, & x \ge 5 \end{cases}$$

Then

   a) Draw the graph for cumulative distribution function.

   b) Write the probability mass function of X. **[3M](CO-2)**

4. The probability density function f of a continuous random variable X is given by:
$$f(x) = \begin{cases} cx + 3, & -3 \leq x \leq -2 \\ 3 - cx, & 2 \leq x \leq 3. \\ 0, & else \end{cases}$$

a) Compute c.

b) Compute the probability $P\left(\frac{5}{2} < X < \frac{11}{4} \mid X > \frac{21}{8}\right)$. **[3M](CO-2)**

5. Certain coded measurements of the pitch diameter of threads of a fitting have the probability density $f(x) = \begin{cases} \frac{4}{\pi(1+x^2)}, & 0 < x < 1 \\ 0, & else \end{cases}$

Find the expected value and variance of this random variable correct up to four decimal places. **[3M](CO-2)**